**Topic:** The Brothers Bernoulli

**Notes on Topic:** The eldest brother, Jakob, was a gifted mathematician who made important contributions to Calculus, the sum of infinite series and to the new subdiscipline, Probability

His masterpiece was *Ars Conjectandi* was a consolidation of his work and earlier works, pushing frontiers in the subject of Probability  
  
Johann was building his own mathematical reputation  
Johann spread Leibniz’s calculus across the continent  
One of Johann’s greatest contributions came through his connection with Marquis de l’Hospital (1661-1704, a French nobleman and amateur mathematician that desperately wanted to learn this new Calculus)  
Marquis then employed Johann to supply him with tracts on the subject and provide him with any new discoveries, thus, l’Hospital essentially bought the rights to Johann’s mathematical research  
In 1696 l’Hospital collected and published Johann’s findings in a book called *Analysis of the Infinitely Small*, the book was exclusively Johann’s, except the name on the title page  
  
In this chapter the great theorem is examined by both Jakob and Johann revolving around the “harmonic series”   
In the seventeenth century, many mathematicians examined the infinite series, and there was never a guarantee that there would be a finite sum, of course, many of these series do not, or we say they diverge to infinity  
In other cases, there are series of infinitely many terms that sum to a finite number  
For example: Leibniz’s series in which sums to 2  
The most important convergence series is the geometric series, a+a^2+a^3+a^4+.... Where -1 < a < 1.   
The “seventeenth century style” argument

then,

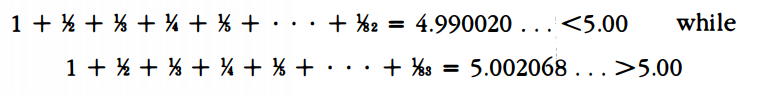
Solving for S leaves,

If we use this same example for a = 2, we see that this formula is absurd and the series could never sum to -2  
So it holds that the absolute value of *a* must be less than one

Then comes the conjecture: a series converges to a finite sum iff the individual terms of the series themselves converge toward zero

This conjecture is only true in the forward implication, in another way, “we cannot hope to get an infinite number of terms to add to a finite sum unless the terms themselves become ever more negligible”

The converse is not true, there are infinite series in which the individual elements tend to zero, but the sum is still diverges to infinity  
  
There is one “pathological counterexample” an example so counterintuitive and bizarre that is is labeled pathological



The sum of the reciprocals of positive integers   
The harmonic series is so unsettling because it takes the first 83 terms to have the sum exceed 5  
And then it takes 144 terms to exceed 6  
To exceed 10 it takes 12,367 terms  
To exceed 20 it takes around a quarter of a billion terms, it seems preposterous that this would ever diverge to infinity, but it does!

**Additional Suggested Reading**: None

**Assignment:** Homework 7 Problem 103